N
530. L 32
4

A SUPPLEMENT TO A Late TREATISE, CALLED *An Eſſay for the Diſcovery of ſome New Geometrical Problems,*

Concerning Angular Sections, reſolving what was there Problematically propoſed; and with ſome Rectification made in the former Eſſay, ſhowing an eaſie method truly Geometrical, without any Conick Section, or Cubick Æquation, to ſect any Angle or Arch of a Circle into 3. 5. 7. or any other Uneven Number of equal parts.

By G. K. K

WHereas it was ſuppoſed in the former Propoſal, that a ſtraight Line could be drawn through the extream Points of three or more Concentrick Arches at both ends (the Arches beginning or ending upon a ſtraight Line coming from the Center of thoſe Concentrick Arches) having equal Cords, though not equal Arches. Upon further conſideration, it is found, that however ſeemingly ſuch a Line may appear to be ſtraight in many caſes, as when the Radius is ſhort, or the Angle very acute, yet in no caſe is ſuch a Line mathematically ſtraight, but is a regular Curve, and can be as regularly drawn, and by as true Geometrical Art, as any Parabola, or other Conick Section can, and with greater facility and readineſs, and which any Tiro who underſtands nothing of Conick Sections, and Cubick Equations may do. The way of drawing the ſaid Curve is this: Let a ſhort croſs-Rule be ſet at right Angles with another longer Rule, and let the length of the croſs Rule be at pleaſure 2 or 3 Inches, or more, as 6 or 7, as ye have a mind to make the length of the Cord of each part of the Section of your Angle, which as in the following Figure let be 3 Inches, and let the juſt half of the croſs-Rule be on the left ſide of the long Rule, and let a ſmall Braſs or Steel-Pin be fixed on the right end of the ſaid croſs Rule, that as the Rule is moved, may make an Impreſſion on the Paper, as the point of the Compaſs doth in drawing a Circle. The length of the longer Rule is to be as occaſion requireth, as double or triple the length of the other.

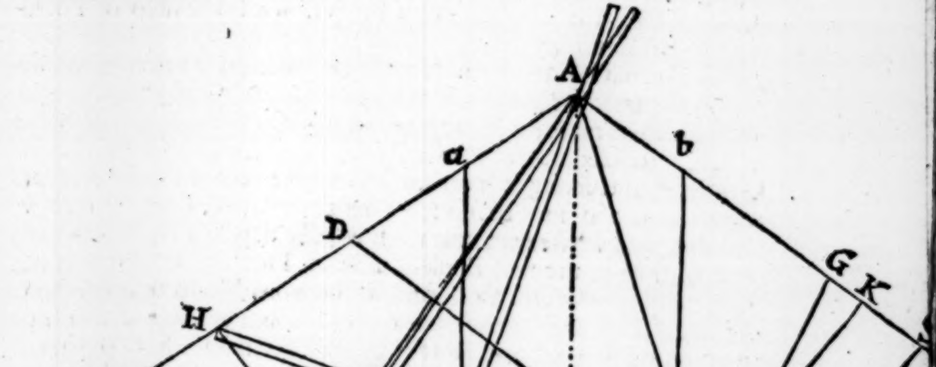
Having thus prepared your two Rules, the one cutting the other at right Angles, and the croſs-Rule fixed to it, (though it may be made alſo moveable on it) ſuppoſe the Angle given to be triſected is BAC, meaſured by the arch BMC. in order to draw the curve Line with one draught of the Hand, ſet the left end of the croſs-Rule, on the point B, and from B let it run or ſlide along the line BA, and as it runs along the ſaid Line, let the left ſide of the long Rule ſtill run through the Center, or vertical point A, which is moſt eaſily done; and let it run or ſlide along from B towards A, until the other end of the croſs-Rule reach at leaſt to the line AC, or further as one pleaſeth, and the Braſs Pin on the other end of the croſs Rule ſhall deſcribe the regular curve FIG Having thus drawn the curve Line, at the diſtance of one half of the croſs Rule draw the ſtraight Line DE paralel to AC. and where the Curve Line cuts the ſtraight line DE as at I, a Line drawn from the center A to I, ſhall by true Geometry, triſect the given angle BAC.

The Demonſtration.

Seeing it is the property of theſe two Rules croſſing each other at right Angles, where ever the two ends of the croſs Rule terminates, to make an Iſoſceles triangle, making always two right angle triangles, whoſe Baſes are equal, and the Perpendicular common to both, therefore by the 4. 1. *el. Eucl.* the Hypotenuſals are equal. Therefore with Radius AI deſcribing the arch HLIS, draw the Cord HI. and from I let fall a perpendicular on the line AC. as IK, making HI=LI=IK, therefore the arches of thoſe equal Sines are equal, as HL=LI=IS. q. e. d. The ſame or any other Angle obtruſe or acute may be triſected into 3 equal parts, without the curve Line, or any part of it, by finding the point I, which can be found without the curve line by letting the croſs Rule ſlide or run along the line AB, (while the left ſide of the long Rule ſtill runneth through the center A) either upwards or downwards, until the right ſide of the croſs Rule touch the

ſtraight

ſtraight line DE which ſhall be at I. And thus without any need of noticing or regarding the Curve Line, the Point I is found, where the two ſtraight lines HI and DE meet together: And as thus any Angle may be triſected without drawing any Curve Line, ſo it may eaſily and truly be done without either Scale or Compaſs, other than what the two croſs Rules are, as any Artiſt may eaſily perceive.



If any object againſt this Method, as Mechanical, and not Mathematical and truly Geometrical, becauſe performed by an Inſtrument, I ſhall refer them to two great Geometricians for its Vindication,

dication, to wit, *DeſCartes* in his ſecond Book of Geometry, and *Franciſcus a Schoten* in his Commentary on him, *argum. lib.* 2. both which do prove that what is performed by Inſtruments Geometrically made, is Geometrical, otherwiſe the plaineſt Geometry muſt be rejected, becauſe its Figures are drawn by Rule and Compaſs, both which are Inſtruments, and not only Parabolas, and other Conick Sections which are Curves, but divers other Curves, yea, all ſuch that can be drawn by Art, with the help of Inſtruments, ſuch as they have deviſed, they contend to be truly Geometrical; and both of them in their Geometrical Treatiſes, uſe divers Inſtruments for deſcribing Curves Geometrically much more difficult to be made, and with more difficulty to be uſed, than what is here propoſed of two ſimple Rules, cutting one another at right Angles. And ſeeing it hath no dependance on Solids, or Algebra Equations, and may be done without any Curve Line, as is above ſhowed, and whoſe demonſtration wholly depends on a few eaſie Propoſitions of the firſt Book of *Euclid*, I ſee not why it may not be called Plain Geometry: And as the word Mechanical is uſed to ſignifie a thing not Mathematically exact, but coming near to it by Approximation, in this ſenſe it is not Mechanical, but Mathematical, and purely Geometrical, being grounded on as good demonſtration, as any Propoſitions in *Euclid*, and being but a Corrolary from ſome of them.

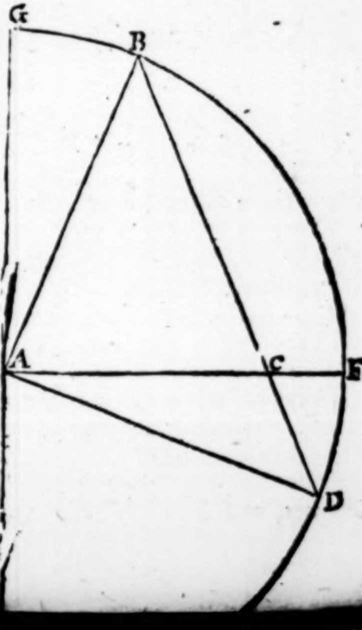
The next thing to be ſhown is the Quinquiſection, where to make one Figure ſerve to both, I make the Croſs Rule only one Inch and one 4th part from the middle line AM, ſetting off on both ſides one half of the length of the croſs Rule, draw the paralel Lines *a d* and *b e*, then let the croſs Rule ſide along the line AB, as in the Triſection, while the left ſide of the long Rule ſlides through the center A, the other end of the croſs Rule ſhall deſcribe a Curve, a part of which ſhall be *g h*, that may be continued at pleaſure. Again, ſetting the right end of the croſs Rule one the Point *d*, let it ſlide or move along the Line *d a*, (while the left ſide of the long Rule runneth through the center A,) the left end of the croſs Rule ſhall deſcribe a part of another Curve, meeting at *h* the other Curve. And having found the point *h* with radius A *h* deſcribe the Arch *v h z x y N* which ſhall give *v h* =one fifth of the whole Arch, as is evident from the foregoing demonſtration.

The Quinquiſection alſo may be made without any Curve, if two long Rules be jointed together like a Sector, and each have a moveable croſs Rule to move on them at right Angles, with the long Rules. For let the center of the two long Rules be fixed on the center A, and let the 2 croſs Rules be moved together from

B and *d*, until (the left end of the one ſtill touching the right end of the other) the right end of the Croſs neareſt to the Line a d touch upon ſome Point of it as at W, the Point at W ſhall give the Quinquiſection as above.

And thus a true Geometrical Line of Cords may be made by any Tiro, without any Conick Section, or Algebra Equation, and without any Table of Natural Sines or Arithmetical Operation; for whereas *Euclid* (11.4.) hath taught how to find the Cord of 36 degr. and alſo it is found by Quinquiſecting the half Circle, as is above ſhewed, it remains only to triſect the Arch of 120 degr. which giveth the Cord of 40, and 36 taken from 40, leaveth 4 degr. which biſected gives 2, and that biſected giveth 1, which is the one 360th part of the Circle, and one 90th of the Quadrant; and this is more methodical than to teach a beginner to make his Line of Cords, for projecting of Angles, by ſending him to Conick Sections, and Algebra Equations or the Table of Natural Sines, which he is not capable at his entry, nor after he has made ſome good progreſs to underſtand (it being to teach *ignotum per ignotius* an unknown thing by a more unknown) quite contrary to all good method of true Science, ſuch as Geometry is. The method of the Quinquiſection here delivered, ſufficiently ſheweth without example, any other Section deſired.

The Corrolaries mentioned in the former Treatiſe, with the Rectification here made, are all valid, ſome of the chief of them I ſhall here mention.



1. One great Uſe is to teach a beginner how to make a true Line of Cords, as is above ſhowed, and how to divide a Circle into any parts required.

2. Another great uſe *Deſcartes* ſhoweth in his third Book of Geometry, for the reſolving any ſuch Equation in Algebra as $z\,3 = + p\; z +- q$, where the Root z is an unknown quantity, and can be found by the Triſection of an Angle.

3. A third great uſe is to give ſome New Promblems in Practical Geometry, one whereof I ſhall here ſhow. Let a ſtraight Line AF be given, (ſee the *ſecond Figure*) and it is required on the point A to erect an Iſoſceles ABC, whoſe ſide BC produced, ſhall terminate on a limited point D, under the given ſtraight line AF.

The conſtruction is thus, draw a ſtraight Line from D to A, as DA, next make the right Angle FAE. Divide the angle EAD into three equal parts, and with radius AD deſcribe the Semicircle GFE. From C to B ſet off GB= ED. Then draw the line AB, and from B draw the line BCD, which ſhall form the Iſoſceles triangle ABC, whoſe ſide BC being produced, ſhall terminate on D. q. e. f. the uſe of this is obvious in Architecture.

4. A fourth great uſe is to give us ſome New Problems in Geography and Navigation.

Example.

There are four places A. B C. D ſo ſituated. A is diſtant from D 100 Leagues, and beareth South-Eaſterly from it 70 degr. B is diſtant from A 100 Leagues South-Weſterly. C is diſtant from B 100 Leagues North-Weſterly C and A are in the ſame Latitude and ſo that theſe three places B. C. D lie in a ſtraight Line one from another.

Q. What is the diſtance betwixt theſe two places, B and D, and the Courſe on the Rhumb Line betwixt A and B, and the diſtance betwixt A and C.

The Reſolution. Divide the angle EAD into three equal parts, and make CAB= one third part of EAD, and draw the line BAD. Thus the four places A. B. C. D ſhall be duly ſituated, and an Iſoſceles Triangle ſhall be formed ABD, whoſe ſide AB=AD=100 Leagues, and the angle BAD=86 degr. 40. conſequently by plain Tri-

Trigonometry, the angle of the courſe GAB being found, which is 23 degr. 30 min. the angle ABD, its double is 46 degr. 40=ADB. by the Rule of Oppoſits.

	Log.
As the Sine of 46 d. 40	9. 861757
to AD 100 Leagues, ſo	2. 000000
the Sine of 86. 40	9. 999265
	11. 999265
	9. 861757
to BD 137. $\frac{1}{4}$ fere	2. 137508

From which ſubſtracting BC 100 Leagues, there remaineth CD 37. $\frac{1}{4}$. as was required.

A Fifth great uſe of the Triſection, and other Sections is, having the Ratio of any 2 Angles given in any plain Triangle, to find the quantity of them, if the quantity of the 3d Angle be given, without any regard to their ſides. What other uſes theſe Angular Sections may have, is left to the ſearch of Induſtrious Artiſts.

London, *Printed for the Author, and are to be had at the* Three Pigeons *over againſt the* Exchange, *and at his Houſe in* Pudding-lane, *at the Sign of the* Golden Ball, *where he Teacheth the Mathematical Arts.*